Math 43 Midterm 1 Review

CONICS

[1] Find the equations of the following conics.
If the equation corresponds to a circle, find its center & radius.
If the equation corresponds to a parabola, find its focus, vertex, directrix & axis of symmetry.
If the equation corresponds to an ellipse, find its center, foci & the endpoints of the major and minor axes.
If the equation corresponds to a hyperbola, find its center, foci, vertices & asymptotes.

[a] Ellipse: foci $(-5, 2)$ and $(3, 2)$, minor axis of length 12
[b] Parabola: vertex $(-4, 6)$, directrix $x = 16$
[c] Hyperbola: one focus $(2, 1)$, asymptotes $y = 3x - 7$ and $y = -3x + 5$
[d] Ellipse: endpoints of major axis $(-7, -3)$ and $(5, -3)$, endpoints of minor axis $(-1, -7)$ and $(-1, 1)$
[e] Hyperbola: vertices $(4, -1)$ and $(-2, -1)$, passing through $(5, 6)$
[f] Parabola: vertex $(-4, -6)$, focus $(-4, -11)$
[g] Ellipse: vertices $(-3, -7)$ and $(-3, 5)$, foci $(-3, -6)$ and $(-3, 4)$
[h] Circle: endpoints of diameter $(-8, 7)$ and $(-2, -3)$
[i] Hyperbola: vertices $(-3, -7)$ and $(-3, 5)$, foci $(-3, -9)$ and $(-3, 7)$
[j] Ellipse: foci $(5, -2)$ and $(5, 4)$, major axis of length 12
[k] Parabola: focus $(4, -6)$, directrix $y = 16$
[l] Ellipse: vertices $(12, 1)$ and $(2, 1)$, minor axis of length 4
[m] Hyperbola: one vertex $(6, 2)$, asymptotes $y = 2x - 4$ and $y = -2x + 8$
[n] Parabola: vertex at origin, horizontal axis of symmetry, passing through $(-8, -7)$
[o] Ellipse: endpoints of minor axis $(3, 11)$ and $(3, 7)$, major axis of length 10

[2] Determine if each equation corresponds to a circle, a parabola, an ellipse or a hyperbola.
If the equation corresponds to a circle, find its center & radius.
If the equation corresponds to a parabola, find its focus, vertex, directrix & axis of symmetry.
If the equation corresponds to an ellipse, find its center, foci & the endpoints of the major and minor axes.
If the equation corresponds to a hyperbola, find its center, foci, vertices & asymptotes.

[a] $9x^2 - 16y^2 - 36x + 32y + 164 = 0$
[b] $y^2 + 8x + 2y - 39 = 0$
[c] $x^2 + y^2 + 6x - 8y + 21 = 0$
[d] $9x^2 + 4y^2 + 72x + 40y + 208 = 0$

POLAR

[3] Remember that a single point in the plane has infinitely many polar co-ordinates.
Consider the point with polar co-ordinates $(7, \frac{2\pi}{3})$.

- [a] Find another pair of polar co-ordinates for this point, using a positive $r-$value, and a negative $\theta-$value.
- [b] Find another pair of polar co-ordinates for this point, using a negative $r-$value, and a positive $\theta-$value.
- [c] Find another pair of polar co-ordinates for this point, using a negative $r-$value, and a negative $\theta-$value.

[4] Convert the following points or equations.

- [a] the point with polar co-ordinates $(8, \frac{5\pi}{6})$ to rectangular co-ordinates
- [b] the points with rectangular co-ordinates $(-6, -2)$ to polar co-ordinates
- [c] the polar equation $r = \cos 2\theta$ to rectangular
- [d] the rectangular equation $x^2 - y^2 - 2x = 0$ to polar

[5] Run the standard tests for symmetry for the polar equation $r^3 = 1 - \sin 2\theta$, and state the conclusions.
What is the minimum interval of $\theta-$values that must be plotted before using symmetry to complete the graph ?

[6] Find the zeros of the polar equation $r = 2\cos 2\theta + 1$.

[7] Determine if each polar equation corresponds to a circle, a parabola, an ellipse or a hyperbola.
If the equation corresponds to a circle, find its center & radius.
If the equation corresponds to a parabola, find its eccentricity, focus, directrix & vertex.
If the equation corresponds to an ellipse, find its eccentricity, foci, directrix, center & the endpoints of the major axes and latera recta.
If the equation corresponds to a hyperbola, find its eccentricity, foci, directrix, center, vertices & the endpoints of the latera recta.
Do not convert the equations to rectangular co-ordinates.
Final answers must be in rectangular co-ordinates.

[a] $r = \dfrac{10}{3 - 3\sin\theta}$ [b] $r = \dfrac{10}{3 - 2\cos\theta}$ [c] $r = \dfrac{10}{2 + 3\sin\theta}$ [d] $r = 10$

[8] Find the polar equations of the following conics with their focus at the pole.

- [a] Parabola: directrix $x = 7$
- [b] Parabola: vertex $(7, \frac{3\pi}{2})$
- [c] Ellipse: eccentricity $\frac{3}{4}$, directrix $y = 5$
- [d] Ellipse: vertices $(4, 0)$ and $(2, \pi)$
- [e] Hyperbola: eccentricity $\frac{5}{2}$, directrix $x = -3$
- [f] Hyperbola: vertices $(3, \frac{3\pi}{2})$ and $(15, \frac{3\pi}{2})$

ANSWERS

CONICS

[1] [a] Center: $(-1, 2)$

Foci: GIVEN

Endpoints of major axis: $(-1 \pm 2\sqrt{13}, 2)$

Endpoints of minor axis: $(-1, -4)$ and $(-1, 8)$

Equation: $\dfrac{(x+1)^2}{52} + \dfrac{(y-2)^2}{36} = 1$

[b] Focus: $(-24, 6)$

Vertex: GIVEN

Directrix: GIVEN

Axis of Symmetry: $y = 6$

Equation: $(y-6)^2 = -80(x+4)$

[c] Center: $(2, -1)$

Foci: $(2, 1)$ and $(2, -3)$

Vertices: $(2, -1 \pm \frac{3\sqrt{10}}{5})$

Asymptotes: GIVEN

Equation: $\dfrac{(y+1)^2}{\frac{18}{5}} - \dfrac{(x-2)^2}{\frac{2}{5}} = 1$

[d] Center: $(-1, -3)$

Foci: $(-1 \pm 2\sqrt{5}, -3)$

Endpoints of major axis: GIVEN

Endpoints of minor axis: GIVEN

Equation: $\dfrac{(x+1)^2}{36} + \dfrac{(y+3)^2}{16} = 1$

[e] Center: $(1, -1)$

Foci: $(1 \pm 6\sqrt{2}, -1)$

Vertices: GIVEN

Asymptotes: $y + 1 = \pm\sqrt{7}(x-1)$

Equation: $\dfrac{(x-1)^2}{9} - \dfrac{(y+1)^2}{63} = 1$

[f] Focus: GIVEN

Vertex: GIVEN

Directrix: $y = -1$

Axis of Symmetry: $x = -4$

Equation: $(x+4)^2 = -20(y+6)$

[g] Center: $(-3, -1)$

Foci: GIVEN

Endpoints of major axis: GIVEN

Endpoints of minor axis: $(-3 \pm \sqrt{11}, -1)$

Equation: $\dfrac{(x+3)^2}{11} + \dfrac{(y+1)^2}{36} = 1$

[h] Center: $(-5, 2)$

Radius: $\sqrt{34}$

Equation: $(x+5)^2 + (y-2)^2 = 34$

[i] Center: $(-3, -1)$
Foci: GIVEN
Vertices: GIVEN
Asymptotes: $y+1 = \pm\frac{3\sqrt{7}}{7}(x+3)$
Equation: $\frac{(y+1)^2}{36} - \frac{(x+3)^2}{28} = 1$

[j] Center: $(5, 1)$
Foci: GIVEN
Endpoints of major axis: $(5, -5)$ and $(5, 7)$
Endpoints of minor axis: $(5 \pm 3\sqrt{3}, 1)$
Equation: $\frac{(x-5)^2}{27} + \frac{(y-1)^2}{36} = 1$

[k] Focus: GIVEN
Vertex: $(4, 5)$
Directrix: GIVEN
Axis of Symmetry: $x = 4$
Equation: $(x-4)^2 = -44(y-5)$

[l] Center: $(7, 1)$
Foci: $(7 \pm \sqrt{21}, 1)$
Endpoints of major axis: GIVEN
Endpoints of minor axis: $(7, -1)$ and $(7, 3)$
Equation: $\frac{(x-7)^2}{25} + \frac{(y-1)^2}{4} = 1$

[m] Center: $(3, 2)$
Foci: $(3 \pm 3\sqrt{5}, 2)$
Vertices: $(0, 2)$ and $(6, 2)$
Asymptotes: GIVEN
Equation: $\frac{(x-3)^2}{9} - \frac{(y-2)^2}{36} = 1$

[n] Focus: $\left(-\frac{49}{32}, 0\right)$
Vertex: GIVEN
Directrix: $x = \frac{49}{32}$
Axis of Symmetry: $y = 0$
Equation: $y^2 = -\frac{49}{8}x$

[o] Center: $(3, 9)$
Foci: $(3 \pm \sqrt{21}, 9)$
Endpoints of major axis: $(8, 9)$ and $(-2, 9)$
Endpoints of minor axis: GIVEN
Equation: $\frac{(x-3)^2}{25} + \frac{(y-9)^2}{4} = 1$

[2] [a] HYPERBOLA

Center: $(2, 1)$

Foci: $(2, -4)$ and $(2, 6)$

Vertices: $(2, -2)$ and $(2, 4)$

Asymptotes: $y - 1 = \pm\frac{3}{4}(x-2)$

[b] PARABOLA

Focus: $(3, -1)$

Vertex: $(5, -1)$

Directrix: $x = 7$

Axis of Symmetry: $y = -1$

[c] CIRCLE

Center: $(-3, 4)$

Radius: 2

[d] ELLIPSE

Center: $(-4, -5)$

Foci: $(-4, -5 \pm \sqrt{5})$

Endpoints of major axis: $(-4, -8)$ and $(-4, -2)$

Endpoints of minor axis: $(-6, -5)$ and $(-2, -5)$

POLAR

[3] [a] $(7, -\frac{4\pi}{3})$ [b] $(-7, \frac{5\pi}{3})$ [c] $(-7, -\frac{\pi}{3})$

[4] [a] $(-4\sqrt{3}, 4)$ [b] $(2\sqrt{10}, 3.46)$

[c] $(x^2 + y^2)^3 = (x^2 - y^2)^2$ [d] $r = \dfrac{2\cos\theta}{\cos 2\theta} = 2\cos\theta \sec 2\theta$

[5] Run the standard tests for symmetry for the polar equation $r^3 = 1 - \sin 2\theta$, and state the conclusions.
What is the minimum interval of $\theta -$ values that must be plotted before using symmetry to complete the graph ?

Symmetry over polar axis:	substituting $(r, -\theta)$	gives $r^3 = 1 + \sin 2\theta$	no conclusion
	substituting $(-r, \pi - \theta)$	gives $r^3 = -1 - \sin 2\theta$	no conclusion
Symmetry over pole:	substituting $(-r, \theta)$	gives $r^3 = -1 + \sin 2\theta$	no conclusion
	substituting $(r, \pi + \theta)$	gives $r^3 = 1 - \sin 2\theta$	symmetric over pole
Symmetry over $\theta = \frac{\pi}{2}$:	substituting $(-r, -\theta)$	gives $r^3 = -1 - \sin 2\theta$	no conclusion
	substituting $(r, \pi - \theta)$	gives $r^3 = 1 + \sin 2\theta$	no conclusion

Minimum interval $\theta \in [0, \pi]$ or $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

[6] $0 = 2\cos 2\theta + 1$ for $0 \le \theta < 2\pi$

$\cos 2\theta = -\frac{1}{2}$

$2\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$ since $0 \le 2\theta < 4\pi$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

[7] [a] PARABOLA

Eccentricity: 1

Focus: $(0, 0)$

Directrix: $y = -\frac{10}{3}$

Vertex: $(0, -\frac{5}{3})$

[b] ELLIPSE

Eccentricity: $\frac{2}{3}$

Foci: $(0, 0)$ and $(8, 0)$

Directrix: $x = -5$

Center: $(4, 0)$

Endpoints of major axis: $(-2, 0)$ and $(10, 0)$

Endpoints of latera recta: $(0, \pm\frac{10}{3})$ and $(8, \pm\frac{10}{3})$

[c] HYPERBOLA

Eccentricity: $\frac{3}{2}$

Foci: $(0, 0)$ and $(0, 12)$

Directrix: $y = \frac{10}{3}$

Center: $(0, 6)$

Vertices: $(0, 2)$ and $(0, 10)$

Endpoints of latera recta: $(\pm 5, 0)$ and $(\pm 5, 12)$

[d] Center: $(0, 0)$

Radius: 10

[8] [a] $r = \dfrac{7}{1+\cos\theta}$ [b] $r = \dfrac{14}{1-\sin\theta}$ [c] $r = \dfrac{15}{4+3\sin\theta}$

[d] $r = \dfrac{8}{3-\cos\theta}$ [e] $r = \dfrac{15}{2-5\cos\theta}$ [f] $r = \dfrac{15}{2-3\sin\theta}$